Thermodynamics of black branes as interacting branes

Shotaro Shiba (KEK, Japan)

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Branes in supergravity

(Low energy effective theory
of string theory)

➢ 4d Einstein gravity

- Blackhole solutions with singularity and horizon

➢ D-dimensional supergravity (usually D≤11)

- Solutions with singularity and horizon
- Singularity is not necessarily point-like, e.g.,

$$ds_D^2 = H^{\frac{p+1}{D-2}} \left[H^{-1}(-f dt^2 + \underline{dy_1^2 + \ldots + dy_p^2}) + f^{-1} dr^2 + r^2 d\Omega_{D-p-2}^2 \right]$$

"black p-brane"

$$H = 1 + \frac{Q}{r^{D-p-3}}, \quad f = 1 - \frac{2\mu}{r^{D-p-3}}.$$

Microscopic description

of black branes from string theory

- D-brane: object with string-scale energy density
- Black (D-)brane: system of infinitely many D-branes

➢ Famous D1-D5-P brane system *[Strominger-Vafa '96]*

- We obtain a blackhole system in 5d spacetime.

	t	1	2	3	4	(5)	T^4
D1-brane	$-$	point-like				$-$	
D5-brane	$-$					$-$	$-$

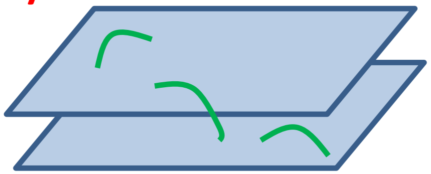
compactified

- Blackhole entropy

$$S = \frac{A}{4G_N}$$ (Bekenstein-Hawking formula)

can be reproduced by counting the number of states of strings.

- State of strings: Open strings ending on branes with energy due to momentum P (= integer/compact radius).
- We can calculate partition function and entropy:
  It is perfectly matched with BH entropy!



In this analysis, <u>all the branes are coincident and static</u>.

- To retain this coincidence, we need to introduce background (NS-NS B-)field. *[David-Mandal-Wadia '02]*
- Some quantities are unchanged due to supersymmetry (non-renormalization theorem).
- But the system with background field is not what we want to see! ← unsatisfactory point

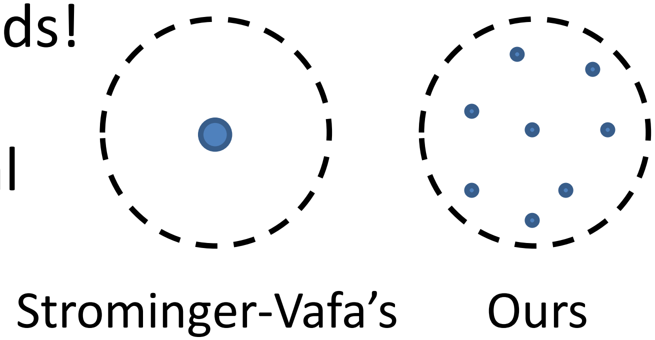
Our new description

[Morita-SS-Wiseman-Withers ’13]

- It may be able to overcome such dissatisfaction.

In our picture, all the branes are separated and moving.

- Separated in the scale of horizon radius
- Moving at the speed ∝ Hawking temperature
- Branes have kinetic energy, so they are gravitationally interacting and compose a bound state (= black brane). We don’t need to put background fields!
- In the following, we neglect numerical factors… ← weak point



Plan of talk

- Introduction (finished)
- Gauge theory approach

  ex. DOFs of M-brane systems
- Gravity theory approach

  ex. intersecting brane systems
- Discussions and Summary

Gauge theory approach

Recipe

1. Obtain an effective action to describe interacting branes, using gauge field theory on a brane system.

2. Evaluate this action in a suitable manner for our picture. “p-soup model”
→ Various physical quantities can be calculated.

3. Compare our results with SUGRA calculations.

Obtain effective action

- We consider black Dp-brane as a simple example.

- Gauge theory is (p+1)-dimensional super Yang-Mills.

$$S_{\mathrm{D}p} = \frac{N}{\lambda}\int d\tau d^p x \,\mathrm{Tr}\left[\frac{1}{4}\mathcal{F}_{\mu\nu}^2 + \frac{1}{2}(D_\mu \Phi^I)^2 + \frac{i}{2}\bar{\Psi}\Gamma^\mu D_\mu \Psi - \frac{1}{4}[\Phi^I, \Phi^J]^2 - \frac{i}{2}\bar{\Psi}\Gamma^I[\Phi^I, \Psi]\right]$$

$$\boxed{\lambda = g_{YM}^2 N}$$

N: rank of gauge group
= # of Dp-branes

- Classical vacua are

$$A_{\mu,ab} = a_{\mu,a}\delta_{ab}\,, \quad \Phi^I_{ab} = \boxed{\phi^I_a}\delta_{ab}\,, \quad \Psi_{ab} = 0$$

(a,b = 1, …, N)

- In the following, we discuss dynamics of scalar moduli.
  They indicate the positions of the branes (in vertical directions).

- We derive the effective action of the scalar moduli.

- Classical part :

$$S^{\text{classical}}_{\text{D}p} = \frac{N}{\lambda}\int d\tau d^p x \sum_a \left(\frac{1}{2}\partial^\mu \phi^I_a \partial_\mu \phi^I_a\right)$$

- 1-loop part :

$$S^{\text{one-loop}}_{\text{D}p,T=0} = -\int d\tau d^p x \sum_{a<b} \frac{\Gamma\left(\frac{7-p}{2}\right)}{(4\pi)^{\frac{1+p}{2}}}\left(2\frac{\{\partial_\mu(\phi^I_a-\phi^I_b)\partial_\nu(\phi^I_a-\phi^I_b)\}^2}{|\phi_a-\phi_b|^{7-p}} - \frac{\{\partial_\mu(\phi^I_a-\phi^I_b)\partial^\mu(\phi^I_a-\phi^I_b)\}^2}{|\phi_a-\phi_b|^{7-p}}\right) + \ldots$$

It starts from $(\partial\phi)^4$ due to supersymmetry.

If branes are static, no potentials exist. (No-force condition)

- Similarly, we can derive more than 1-loop parts (in principle).

➢ 1-loop part (more precisely…)

$$S_{\mathrm{D}p}^{\text{one-loop}} = \frac{1}{8(7-p)(2\pi)^{p-4}\Omega_{8-p}}$$

$$\times \int dt d^p x \sum_{a<b} \frac{2\left(\partial_\mu \vec{\phi}_{ab} \cdot \partial_\nu \vec{\phi}_{ab}\right)^2 - \left(\partial_\mu \vec{\phi}_{ab} \cdot \partial^\mu \vec{\phi}_{ab}\right)^2}{\left|\vec{\phi}_{ab}\right|^{7-p}} \left(\boxed{1} + O\left(\frac{(\partial\phi_{ab})^2}{\phi_{ab}^4}\right)\right)$$

higher derivative terms

$$-\frac{16}{(2\pi)^{p/2}} \int d\tau d^p x \sum_{a<b} \frac{U_a U_b^\star + U_b U_a^\star}{\beta^{1+p}} e^{-\beta|\vec{\phi}_{ab}|} \left(\beta|\vec{\phi}_{ab}|\right)^{p/2}$$

temperature dependent terms

- In the following, we consider only the first term.
- Higher derivative terms and temperature dependent terms can be neglected in our setting, as we will see soon.

(U_a : a Polyakov loop in the Euclidean time direction)

Evaluate effective action

[Wiseman '13]

- To evaluate the effective action in our picture, we impose the following three settings.

1. We set characteristic scale of the brane system.

$$\boxed{\phi_a^I \sim \phi_a^I - \phi_b^I \sim \phi}$$

→ horizon radius

2. We set the relation of "velocity" and temperature.

$$\boxed{\partial\phi_a^I \sim \partial(\phi_a^I - \phi_b^I) \sim \frac{\pi}{\beta}\phi}$$

- Because a <u>thermal</u> field can be expanded in Matsubara modes.
- Then the separated branes satisfy…

$$\phi = \sum_n \phi_n \exp\left(i\frac{2\pi n}{\beta}t\right)$$

$$\beta|\phi| \gg 1 \Leftrightarrow |\partial\phi/\phi^2| \ll 1$$

3. We impose a kind of the virial theorem.

total action

$$\boxed{S_{\mathrm{D}p}^{\text{classical}} \sim S_{\mathrm{D}p,T=0}^{\text{one-loop}} \sim S_{\mathrm{D}p}}$$

- Because we consider a system confined in a finite region.
- It says all the loop contributions are of the same order.
  We look at strong coupling (non-perturbative) region.

➢ We call this setting "p-soup model."

- Dp-branes are scattered but interacting strongly. (like liquid)
- The effective action can be evaluated as

$$\int d\tau \sim \beta\,,\ \sum_a \sim N$$

$$S_{\mathrm{D}p}^{\text{classical}} \sim \int d^p x \frac{N^2}{\beta\lambda}\phi^2\,, \quad S_{\mathrm{D}p,T=0}^{\text{one-loop}} \sim \int d^p x \frac{N^2}{\beta^3 \phi^{3-p}}$$

$$\boxed{F_{\mathrm{D}p} \sim S_{\mathrm{D}p}/\beta \sim N^2 T^{\frac{2(7-p)}{5-p}} \lambda^{-\frac{3-p}{5-p}} V_p\,, \quad \phi \sim T^{\frac{2}{5-p}} \lambda^{\frac{1}{5-p}}}$$

SUGRA results are reproduced!

= horizon radius

Application to M-branes

[Morita-SS ’13]

- We can discuss black M-branes in a parallel way.

- Gauge theory on M2-branes: ABJM theory
- Gauge theory on M5-branes:
  Not formulated yet. But if we assume that scalar moduli exist, dimensional analysis can be done.
- Maybe we can also discuss black branes in F-theory…

[Morita-SS ’15]

➢ Black M2-brane: Analysis using ABJM theory

$$S_{\rm ABJM} = \frac{k}{2\pi}\int d\tau d^2x \, \Big(\mathrm{Tr}\left[(D_\mu \Phi^\dagger_A)(D^\mu \Phi^A) + i\Psi^{\dagger A}\gamma^\mu D_\mu \Psi_A\right] + \mathcal{L}^{(1)}_{\rm CS} - \mathcal{L}^{(2)}_{\rm CS} - V_B - V_F\Big)\,,$$

k: level of Chern-Simons term

- Effective action

$$S^{\rm classical}_{\rm ABJM} = \frac{k}{2\pi}\int d\tau d^2x \sum_a \left(\frac{1}{2}\partial^\mu \phi^I_a \partial_\mu \phi^I_a\right)$$

$$S^{\text{one-loop}}_{{\rm ABJM},T=0} \sim -\int d\tau d^2x \sum_{a<b}\left(\frac{\{\partial(\phi_a-\phi_b)\partial(\phi_a-\phi_b)\}^2}{|\phi_a-\phi_b|^6}\right) + \ldots$$

- Evaluation by p-soup model

$$S^{\rm classical}_{\rm ABJM} \sim \int d^2x \frac{kN}{\beta}\phi^2\,, \quad S^{\text{one-loop}}_{{\rm ABJM},T=0} \sim \int d^2x \frac{N^2}{\beta^3\phi^2}$$

- SUGRA results can be reproduced.

$$\boxed{F_{\rm ABJM} \sim S_{\rm ABJM}/\beta \sim N^{\frac{3}{2}}\sqrt{k}T^3 V_2\,, \quad \phi \sim \frac{N^{\frac{1}{4}}}{k^{\frac{1}{4}}\beta^{\frac{1}{2}}}}$$

Nontrivial N-dependence is reproduced!

➢ Black M5-brane: Dimensional analysis

- Effective action

$$S_{\mathrm{M5}}^{\mathrm{classical}} \sim \int d\tau d^5x \sum_a \left(\frac{1}{2}\partial^\mu \phi_a^I \partial_\mu \phi_a^I\right)$$

: The mass dimension of ϕ is 2.

$$S_{\mathrm{M5},T=0}^{\text{one-loop}} \sim -\int d\tau d^5x \sum_{a<b} \left(\frac{\{\partial(\phi_a-\phi_b)\partial(\phi_a-\phi_b)\}^2}{|\phi_a-\phi_b|^3}\right) + \dots$$

- Evaluation by p-soup model

$$S_{\mathrm{M5}}^{\mathrm{classical}} \sim \int d^5x \frac{N}{\beta}\phi^2 , \quad S_{\mathrm{M5},T=0}^{\text{one-loop}} \sim \int d^5x \frac{N^2\phi}{\beta^3}$$

- SUGRA results can be reproduced.

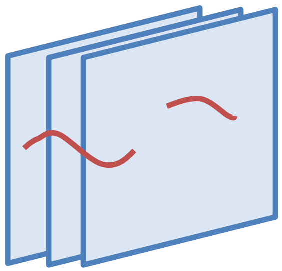
$$F_{\mathrm{M5}} \sim S_{\mathrm{M5}}/\beta \sim N^3 T^6 V_5 \qquad \phi \sim \frac{N}{\beta^2}$$

➢ DOFs of M-branes have nontrivial dependence on N…

[Klebanov-Tseytlin '96]

- D-brane system:

  Strings ending on branes make DOFs. A string has two ends, so DOFs are $O(N^2)$. ← simple explanation!



- M-brane system:

  No strings exist in M-theory.

  DOFs are $O(N^{3/2})$ for M2-branes, and $O(N^3)$ for M5-branes.

- ABJM theory can explain $O(N^{3/2})$ DOFs using the localization technique. But the origin of DOFs is still unclear. *[Marino '11]*

- In our discussion, dynamics of N scalar moduli on branes is the origin of DOFs in both D-brane and M-brane systems!

Gravity theory approach

Recipe

1. Obtain an effective action to describe interacting branes, using probe brane actions in gravity theory.

2. Evaluate this action in "p-soup model."
   → Various physical quantities can be calculated.

3. Compare our results with SUGRA calculations.

Relation of two approaches

[Morita-SS-Wiseman-Withers ’13]

- A kind of gauge/gravity correspondence

- For example, black D-branes can be discussed in both ways.
  (If parallel and one species of branes compose them.)
- Then both approaches give the same effective action.
  After evaluation by p-soup model, we obtain the same results.
- But it is not interesting to consider the same system again.
  So we will see another system that cannot be studied in the gauge approach.

Application to intersecting branes

[Morita-SS ’14, ’15]

- We consider (vertically) intersecting black brane systems. Generally it is difficult to specify the gauge theory on them.
- Then we choose one of the branes in the system as a probe, and consider its <u>probe brane action</u>.
- We assume the background for the probe is the black brane solution itself, since infinitely many branes exist.

$$ds_D^2 = \prod_A H_A^{\frac{q_A+1}{D-2}} \left[-\prod_A \underline{H_A^{-1}} K^{-1} dt^2 + \prod_A \underline{H_A^{-1}} K d\hat{y}_1^2 + \sum_{\alpha=2}^{D-d-3} \prod_A \underline{H_A^{-\delta_A^{(\alpha)}}} dy_\alpha^2 + \sum_{i=1}^{d+2} dx_i^2 \right]$$

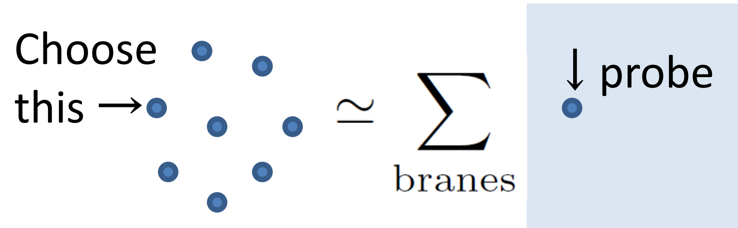
$$H_A = 1 + \frac{\mathcal{Q}_A}{r^d}, \quad K = 1 + \frac{\mathcal{P}}{r^d}$$

Obtain effective actions

- Ex.) intersecting D1-D5 system

	t	1	2	3	4	(5)	T^4
Q_1 D1-brane	$-$					$-$	
Q_5 D5-brane	$-$					$-$	$-$

compactified



- When we choose a D1-brane as a probe, its probe brane (DBI) action on the black brane background is $\quad Q_1,\ Q_5 \gg 1$

$$S_{\rm D1}^{\rm probe} = -m_1 \int dt \left(\frac{1}{H_1}\sqrt{1 - H_1 H_5 \vec{v}^2} - \left(\frac{1}{H_1} - 1 \right) \right),$$

$$H_1 = 1 + \frac{r_1^2}{\vec{r}^2}, \quad H_5 = 1 + \frac{r_5^2}{\vec{r}^2}, \quad r_1^2 = \frac{4 m_1 G_5 Q_1}{\pi}, \quad r_5^2 = \frac{4 m_5 G_5 Q_5}{\pi}.$$

- Expand it for small gravity coupling G_5 and small curvature of branes. (SUGRA description is valid.) $\quad v = \partial_t r \ll 1$

- The condition $v = \partial_t r \ll 1$ means the low energy region, so it corresponds to the near-extremal region $r^2 \ll r_1^2, r_5^2$. Then the dominant terms are

D1-D5 D1-D1-D5-D5

$$S^{\text{probe}}_{\underline{\text{D1}}} = \int dt \left[-m_1 + \frac{m_1}{2}\vec{v}^2 + \underbrace{\frac{m_1}{2}\frac{r_5^2}{r^2}\vec{v}^2}_{\propto G_5 Q_5} + \frac{m_1}{8}\vec{v}^4 + \frac{m_1}{8}\frac{r_1^2}{r^2}\vec{v}^4 + \underbrace{\frac{m_1}{8}\frac{r_1^2 r_5^4}{r^6}\vec{v}^4}_{\propto G_5{}^3 Q_1 Q_5{}^2} + \cdots \right]$$

$$S^{\text{probe}}_{\underline{\text{D5}}} = \int dt \left[-m_5 + \frac{m_5}{2}\vec{v}^2 + \underbrace{\frac{m_5}{2}\frac{r_1^2}{r^2}\vec{v}^2}_{\propto G_5 Q_1} + \frac{m_5}{8}\vec{v}^4 + \frac{m_5}{8}\frac{r_5^2}{r^2}\vec{v}^4 + \underbrace{\frac{m_5}{8}\frac{r_5^2 r_1^4}{r^6}\vec{v}^4}_{\propto G_5{}^3 Q_1{}^2 Q_5} + \cdots \right]$$

- Then we can read off the (dominant) interactions of graviton exchange among all the branes in the system.
- We can write down the effective action as

$$S_{\text{D1D5}} = \int dt \sum_{n=1}^{\infty} L_n, \quad L_n \sim \sum_{i_1,\ldots,i_n}^{Q_1} \sum_{j_1,\ldots,j_n}^{Q_5} \left(G_5^{2n-1} \frac{m_1^n m_5^n}{\pi^{2n-1}} \prod_{k=2}^{n} \prod_{l=1}^{n} \frac{1}{\vec{r}_{i_1 i_k}^{\,2} \vec{r}_{i_1 j_l}^{\,2}} \vec{v}^{2n} + \cdots \right)$$

(2n-1)-graviton exchange interactions among n D1 and n D5-branes

Evaluate effective actions

- Characteristic scale: $\vec{r}_i - \vec{r}_j \sim r\,, \quad \vec{v}_i - \vec{v}_j \sim v\,.$
- Virial theorem (strong coupling) : $L_1 \sim L_2 \sim \cdots \sim \sum_n L_n$
- Relation of velocity and temperature: $v \sim \pi T r$

➢ Then physical quantities can be evaluated as

$$F \sim L_1 \sim \frac{\pi r^2}{G_5}$$

$$r \sim T G_5 \sqrt{Q_1 Q_5 m_1 m_5}$$

$$S_{\text{entropy}} = -\frac{\partial F}{\partial T} \sim \pi m_1 m_5 Q_1 Q_5 G_5 T$$

$$F = -\frac{\pi r_H^2}{8G_5}$$

They reproduce SUGRA results!

$$r_H = 8 G_5 T \sqrt{m_1 m_5 Q_1 Q_5}$$

$$S_{\text{entropy}} = 16\pi m_1 m_5 G_5 Q_1 Q_5 T$$

- This discussion can be applied to all Einstein-Maxwell-Dilaton systems in general dimensions.

Discussions and Summary

Other applications

- Gregory-Laflamme phase transition

- Phase transition of black brane occurs in a compactified space. We can discuss it in our picture. *[Morita-SS-Wiseman-Withers, ‘15]*

- Time-dependent brane system

- Black brane solutions can be generalized by making Harmonic functions time-dependent. We can also discuss its dynamics. *[SS, in progress]*

- Other applications…?

Summary

- The "p-soup model" may be a new picture of black branes.
  In this model, the branes are separated and moving.

- This model can explain the dynamics of various black branes (parallel/intersecting D/M-branes) in a unified manner.

- This model has other strong points:
  We don't need to introduce any extra field. (e.g. NS-NS B-field)
  We don't speculate any UV structure of gravity.

- But this model has a weak point:
  We cannot reproduce numerical (rational) factors…